Direct Adaptive NN Control of MIMO Nonlinear Discrete-Time Systems using Discrete Nussbaum Gain

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Introduction

In this work, direct adaptive neural network (NN) control has been studied for a class of MIMO NARMAX system.

(i) Implicit function based adaptive NN control has been exploited to treat the nonaffine appearance of control input.

(ii) With an assumption on the inverse control gain matrix, an update law using discrete Nussbaum gain and deadzone has been proposed for the MIMO system NN weights adaptation.

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1. Introduction
2. System Representation
3. Preliminaries
4. Direct Adaptive NN Control Design
5. Stability Analysis
6. Simulation

System Representation

Consider a p-input and p-output NARMAX system:

$$y(k+\tau)=F(Y(k),U_{k-1}(k),u(k),D_{k-1}(k),\bar{d}(k))+d(k+\tau-1) \qquad (1)$$

where $\tau \geq 1$, $F(\cdot) \in R^p$ is an unknown smooth vector valued function, $d(k) = [d_1(k), \cdots, d_p(k)]^T \in R^p$ is the bounded disturbance, i.e., $\|d(k)\| \leq d_b$, and

$$\begin{aligned}
Y(k) &= [y_1(k), \cdots, y_1(k-n_1+1), y_2(k), \cdots, \\
&\quad y_2(k-n_2+1), \cdots, y_p(k), \cdots, y_p(k-n_p+1)]^T \\
U_{k-1}(k) &= [u_1(k-1), \cdots, u_1(k-m_1), u_2(k-1), \cdots, \\
&\quad u_2(k-m_2), \cdots, u_p(k-1), \cdots, u_p(k-m_p)]^T \\
D_{k-1}(k) &= [d_1(k-1), \cdots, d_1(k-t_1+1), d_2(k-1), \cdots, \\
&\quad d_2(k-t_2+1), \cdots, d_p(k-1), \cdots, d_p(k-t_p+1)]^T \\
\bar{d}(k) &= [d(k+\tau-2), \cdots, d(k)]^T, \quad \text{if} \quad \tau \geq 2
\end{aligned}$$

Assumptions

Assumption

The system function $F(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k))$ satisfies Lipschitz condition with respect to $D_{k-1}(k)$ and $\bar{d}(k)$.

Assumption

The control gain matrix $G(k) = \frac{\partial F(\cdot)}{\partial u(k)}, \forall k \geq 0$, is a full rank matrix, and its inverse, $G^{-1}(k)$, has an either positive definite or negative definite symmetric part, $G_{IS}(k) = \frac{G^{-1}(k)+G^{-T}(k)}{2}$. In addition, the eigenvalues of $G_{IS}(k)$ are assumed to be bounded.

Control objective

Remark

Assumption 2 relaxes the assumption in earlier work [Ge, 04], which requires the existence of an orthogonal matrix $Q(k)$ multiplying $G^{-1}(k)$ to guarantee the eigenvalues of the product matrix are all positive.

The control objective is to design a control input $u(k)$, such that the system output tracks the bounded desired trajectory $y_d(k) = [y_{d1}(k), \cdots, y_{dp}(k)]^T \in R^p$, while all the closed loop signals remain bounded.

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Implicit function adaptive control

Implicit function based adaptive control was first proposed in [Goh and Lee, 94], where implicit function theorem is used to identify the existence of an ideal control.

Remark

(i) *To use implicit function theorem, the control gain matrix is assumed to be nonsingular everywhere such that the implicit functions at all points can be strung together to furnish a unique implicit function defined globally.*

(ii) *Implicit function control law generally cannot be expressed explicitly, but NN can be used to approximate it.*

Discrete Nussbaum Gain

The discrete Nussbaum gain was proposed in [Lee, 86]. For a discrete sequence $\{x(k)\}$ defined as

$$x(0) = 0, x(k) \geq 0, |\Delta x(k)| = |x(k+1) - x(k)| \leq \delta_0 \qquad (2)$$

where δ_0 can be positive constant, the discrete Nussbaum gain $N(x(k))$ is defined on the sequence $x(k)$ as

$$N(x(k)) = x_s(k) s_N(x(k)) \qquad (3)$$

where $x_s(k)$ is defined as

$$x_s(k) = \sup_{\sigma \leq k} \{x(\sigma)\} \qquad (4)$$

Discrete Nussbaum Gain

The sign function $s_N(x(k))$ is chosen as:
Step (a): At $k = k_1$, measure output $y(k_1)$ and compute $\Delta x(k_1)$ and $x(k_1+1)$ and $S_N(x(k_1))$.
Case $(s_N(x(k_1)) = +1)$:

$$\begin{cases} \text{If} \quad S_N(x(k_1)) \le x_s^{\frac{3}{2}}(k_1), & \text{then go to Step (b)} \\ \text{If} \quad S_N(x(k_1)) > x_s^{\frac{3}{2}}(k_1), & \text{then go to Step (c)} \end{cases}$$

Case $(s_N(x(k_1)) = -1)$:

$$\begin{cases} \text{If} \quad S_N(x(k_1)) < -x_s^{\frac{3}{2}}(k_1), & \text{then go to Step (b)} \\ \text{If} \quad S_N(x(k_1)) \ge -x_s^{\frac{3}{2}}(k_1), & \text{then go to Step (c)} \end{cases}$$

Step (b): Set $s_N(x(k_1+1)) = 1$, go to Step (d).
Step (c): Set $s_N(x(k_1+1)) = -1$, go to Step (d).
Step (d): Return to Step (a) and wait for the measurement of output $y(k_1+1)$.

Stability lemma

Lemma

Consider the following summation

$$S'_N(x(k)) = \sum_{\sigma=0}^{\sigma=k} g(\sigma)N(x(\sigma))\Delta x(\sigma) \qquad (5)$$

where $\Delta x(k) \geq 0$ *and* $g_1 \leq |g(k)| \leq g_2$ *with finite* g_1 *and* g_2*.*

(i) *If* $x(k)$ *increases without bound, then*

$$\sup_{x(k)\geq\delta_0} \frac{1}{x(k)} S'_N(x(k)) = +\infty \quad \inf_{x(k)\geq\delta_0} \frac{1}{x(k)} S'_N(x(k)) = -\infty$$

(ii) *If* $x(k) \leq \delta_1$*, then* $|S'_N(x(k))| \leq \delta_2$*, where* δ_1 *and* δ_2 *are some positive constants.*

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Tracking error

Define error vector
$e(k) = y(k) - y_d(k) = [e_1(k), e_2(k), \ldots, e_p(k)]^T$. It can be written as

$$\begin{aligned} e(k+\tau) &= F(Y(k), U_{k-1}(k), u(k), 0, 0) - y_d(k+\tau) \\ &\quad + \Delta F(k) + d(k+\tau-1) \end{aligned} \tag{6}$$

where

$$\Delta F(k) = F(Y(k), U_{k-1}(k), u(k), D_{k-1}(k), \bar{d}(k))$$

$$- F(Y(k), U_{k-1}(k), u(k), 0, 0)$$

is bounded due the Lipschitz condition of $F(\cdot)$ and boundedness of $\bar{d}(k)$ and $D_{k-1}(k)$.

Ideal control

According to implicit function theorem, there exists a unique and continuous ideal control input:
$u^*(k) = \alpha^c(Y(k), U_{k-1}(k), y_d(k+\tau))$, where $\alpha^c(\cdot)$ is an implicit function such that

$$F(Y(k), U_{k-1}(k), u^*(k), 0, 0) - y_d(k+\tau) = 0 \tag{7}$$

Assumption

Given a bounded output $y(k) \in \Omega_y \subset R^p$, $\forall k > 0$, where Ω_y can be any bounded compact set, there is a corresponding bounded compact set Ω_{u^*} such that the desired control $u^*(k)$ is within the compact set Ω_{u^*}.

NN approximation

Consider employing a linear parametrized high order neural network (HONN) to approximate $u^*(k)$ as follows

$$u^*(k) = W^{*T} S(\bar{z}(k)) + \mu(k) \tag{8}$$

where $W^{*T} \in R^{l \times q}$ is the ideal NN weights matrix, $\bar{z}(k) = [Y^T(k), U_{k-1}^T(k), y_d^T(k+\tau)]^T \in \Omega_z \subset R^q$ with $q = \sum_{i=1}^{p}(n_i + m_i + 1)$ and $\mu(k)$ is the bounded NN approximation error vector satisfying $\|\mu(k)\| \leq \mu^*$.

Details of HONN can be found in [Ge, 01]

NN adaptive control

Then, the adaptive NN control $u(k)$ is constructed as

$$u(k) = \hat{W}^T(k)S(\bar{z}(k)) \tag{9}$$

where $\hat{W}(k) \in R^{l\times q}$ and $S(\bar{z}(k)) \in R^l$. The NN weights adaptation law is given as

$$\hat{W}(k)=\hat{W}(k-\tau)-\gamma N(x(k))S(\bar{z}(k-\tau))a(k)e^T(k)/D(k)$$
$$\Delta x(k)=a(k)\gamma e^T(k)e(k)/D(k), x(0)=0$$
$$D(k)=(1+|N(x(k))|^2)(1+\|S(\bar{z}(k-\tau))\|^2+\|e(k)\|^2)$$
$$a(k)=\begin{cases} 1, & \text{if } \|e(k)\|/(1+|N(x(k))|) > \lambda \\ 0, & \text{otherwise} \end{cases}$$

where $\gamma > 0$ and $\lambda > 0$ can be arbitrary positive constants.

Main result

Remark

In the deadzone design, we do not need to know the upper bounds of the disturbance and NN approximation error. Parameter λ can be any positive constant specified by the designer.

Theorem

All closed-loop signals are guaranteed to be semi global uniformly ultimate bounded, the discrete Nussbaum gain $N(x(k))$ will converge to a constant, and the tracking error satisfies $\lim_{k\to\infty}\sup\|e(k)\| < C\lambda$, *with* $C = \lim_{k\to\infty}(1 + |N(x(k))|)$.

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Tracing error

Using mean value theorem, the tracking error (6) can be written as

$$\begin{aligned} e(k+\tau) &= F(Y(k), U_{k-1}(k), u^*(k), 0, 0) - y_d(k+\tau) \\ &\quad + \Delta F(k) + G_\xi(k)[u(k) - u^*(k)] + d(k+\tau-1) \end{aligned} \quad (10)$$

where $G_\xi(k) = \frac{\partial F(\cdot)}{\partial u(k)}\Big|_{u_\xi(k)}$, $u_\xi(k)$ is a point of line $L(u(k), u^*(k)) = \{\xi \mid \xi = \theta u(k) + (1-\theta)u^*(k),\ 0 \le \theta \le 1\}$.
Using the adaptive NN control (9), we have

$$\tilde{W}^T(k-\tau)S(\bar{z}(k-\tau)) = G_\xi^{-1}(k-\tau)e(k) + d^*(k-1) \quad (11)$$

where $d^*(k-1) = -G_\xi^{-1}(k-\tau)[\Delta F(k-\tau) + d(k-1)] + \mu(k)$ is bounded, i.e., $\|d^*(k-1)\| \le d_b^*$, with d_b^* an unknown constant.

According to Assumption 2 on the inverse control gain matrix, there exist two positive constants $\bar{g}$ and $\underline{g}$ such that

$$\underline{g} I \leq \frac{1}{2}(G_\xi^{-1}(k) + G_\xi^{-T}(k)) \leq \bar{g} I \tag{12}$$

or

$$-\bar{g} I \leq \frac{1}{2}(G_\xi^{-1}(k) + G_\xi^{-T}(k)) \leq -\underline{g} I \tag{13}$$

where I is the identity matrix. It implies there exists a sequence $g(k)$ satisfying $\underline{g} \leq |g(k)| \leq \bar{g}$ such that

$$\begin{aligned} e^T(k) G_\xi^{-1}(k-\tau) e(k) &= e^T(k) \tfrac{G_\xi^{-1}(k-\tau) + G_\xi^{-T}(k-\tau)}{2} e(k) \\ &= g(k) e^T(k) e(k) \end{aligned} \tag{14}$$

Stability proof

Let us consider a positive definite function

$$V(k) = \sum_{j=1}^{\tau} \mathrm{tr}\{\tilde{W}^T(k-\tau+j)\tilde{W}(k-\tau+j)\} \qquad (15)$$

It can be shown that

$$\begin{aligned} &\Delta V(k) = V(k) - V(k-1) \\ &= a(k)\gamma^2 N^2(x(k))\frac{S^T(\bar{z}(k-\tau))S(\bar{z}(k-\tau))e^T(k)e(k)}{D^2(k)} \\ &-2\gamma a(k)\left[g(k)N(x(k))\frac{e^T(k)e(k)}{D(k)} + \frac{N(x(k))e^T(k)d^*(k-1)}{D(k)}\right] \end{aligned}$$

Stability proof

From the definition of $a(k)$ for the deadzone, it can be easily derived that

$$|a(k)N(x(k))e^T(k)d^*(k-1)| \leq a(k)\frac{d_b^*}{\lambda}e^T(k)e(k) \qquad (16)$$

which leads to

$$\Delta V(k) \leq c_1 \frac{a(k)\gamma e^T(k)e(k)}{D(k)} - 2g(k)N(x(k))\frac{\gamma a(k)e^T(k)e(k)}{D(k)}$$

with $c_1 = \gamma + 2d_b^*/\lambda$. It is equivalent to

$$\Delta V(k) \leq c_1 \Delta x(k) - 2g(k)N(x(k))\Delta x(k) \qquad (17)$$

Stability proof

Taking summation and dividing by $x(k)$ on both hand sides of (17) and noting $\Delta x(k) \geq 0$, we have

$$0 \leq \frac{V(k)}{x(k)} \leq c_1 - \frac{2}{x(k)} \sum_{\sigma=1}^{k} g(\sigma) N(x(\sigma)) \Delta x(\sigma) + \frac{c_2}{x(k)}$$

where c_2 is a finite constant.
Then, according to conclusion (i) in Lemma 1, it will yields a contradiction if $x(k)$ is unbounded because a positive function cannot oscillate between infinity and minus infinity. Then, together with (ii) in Lemma 1, we have (i) $x(k)$, $V(k)$ and $N(x(k))$ are bounded, and it implies (ii) $\hat{W}(k)$ is bounded.

Stability proof

The boundedness of $x(k)$ implies that $\lim_{k\to\infty} \Delta x(k) = 0$. It implies
$\lim_{k\to\infty} a(k) = 0$ or $\lim_{k\to\infty} \frac{e^T(k)e(k)}{D(k)} \to 0$

which leads to

$\lim_{k\to\infty} \sup \frac{\|e(k)\|}{1+|N(x(k))|} \leq \lambda$ or
$\lim_{k\to\infty} \sup \|e(k)\| \to 0$ according to Key Technical Lemma.

Denote $C = \lim_{k\to\infty}(1 + |N(x(k))|)$, we have
$\lim_{k\to\infty} \sup \|e(k)\| < C\lambda$
Then the boundedness of outputs $y(k)$ is obvious. The boundedness of $\hat{W}(k)$ leads to the boundedness of $u(k)$.

Outline

Simulation model

The following MIMO NARMA system is used for simulation:

$$\begin{aligned} y_1(k+2) &= \pm 0.2\sin(u_1(k)) \pm u_1(k) + d_1(k) \\ &+ \frac{0.6\cos(y_2(k-1)) + y_1(k-1) + 1.2u_2(k) + y_2(k)u_2(k-1)}{1 + y_2^2(k) + y_1^2(k-1) + 3u_2^2(k-1)} \\ y_2(k+2) &= \pm\cos(u_2(k)) \pm 0.5u_2(k) + d_2(k) \\ &+ 1.1\frac{y_2(k-1) + 1.6\sin(y_1(k))u_1(k-1)}{1 + u_1^2(k-1) + 2y_2^2(k-1)} \end{aligned} \tag{18}$$

where $d_1(k) = 0.01\cos(0.1k)$ and $d_2(k) = 0.01\sin(0.1k)$ are disturbances.

The reference trajectories are:

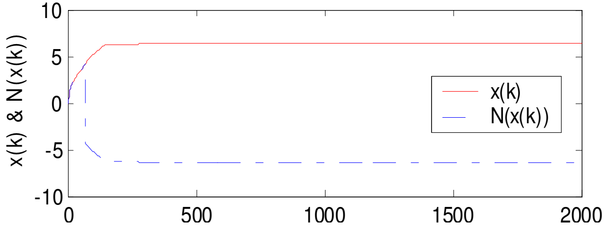
$y_{d1}(k) = 0.5 + 0.25\cos(0.25\pi Tk) + 0.25\sin(0.5\pi Tk)$ and $y_{d2}(k) = 0.5 + 0.25\sin(0.25\pi Tk) + 0.25\sin(0.5\pi Tk)$, with $T = 0.01$, respectively.

Simulation result

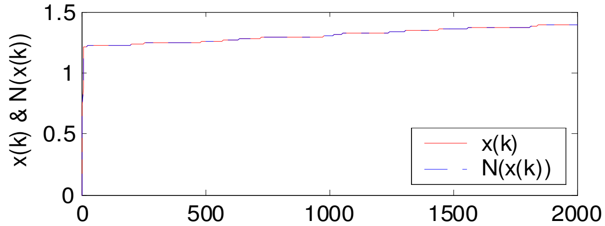
The simulation has been done twice with control fixed, but the "$\pm$" in the simulation model has been chosen to be "$+$" and "$-$", respectively, such that such that the inverse control gain matrix $G^{-1}(k)$ has an either positive definite or negative definite symmetric part.

In the simulation, it can be seen that in both cases, the proposed adaptive NN control works well. The discrete Nussbaum gain $N(x(k))$ adapts by searching alternately in the two directions. It can been see that it turns from positive to negative when the model is with negative sign "$-$".

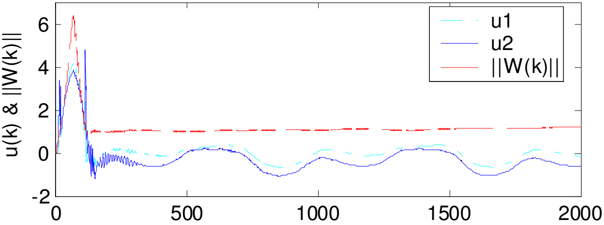
Detail discussion of the Discrete Nussbaum gain can be found in [Lee, 87].



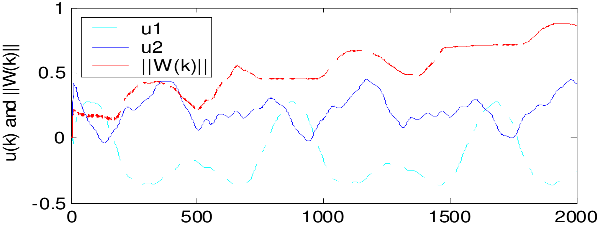
(a) Discrete Nussbaum gain $N(k)$ and $x(k)$(negative sign)



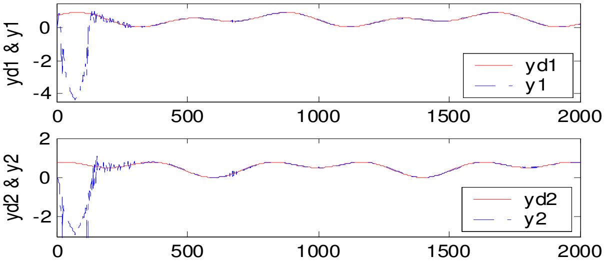
(b) Discrete Nussbaum gain $N(k)$ and $x(k)$(positive sign)



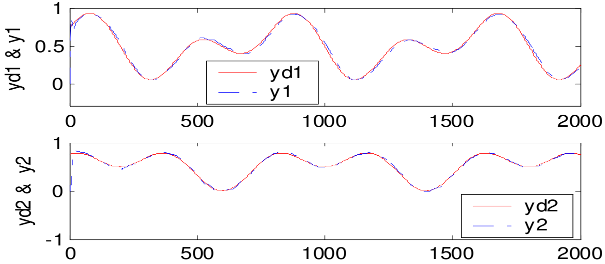
(c) Control signal and NN weights norm (negative sign)



(d) Control signal and NN weights norm (positive sign)



(e) Output tracking(negative sign)



(f) Output tracking (positive sign)

Thank You Very Much!!

Q&A